Unit Introduction

Variables and Patterns

Introducing Algebra

Goals of the Unit

- Identify quantitative variables in situations
- Recognize situations where changes in variables are related in useful patterns
- Describe patterns of change shown in words, tables, and graphs of data
- Construct tables and graphs to display relations among variables
- Observe relationships between two quantitative variables as shown in a table, graph, or equation and describe how the relationship can be seen in each of the other forms of representation
- Use algebraic symbols to write rules and equations relating variables
- Use tables, graphs, and equations to solve problems
- Use graphing calculators to construct tables and graphs of relations between variables and to answer questions about these relations

Developing Students' Mathematical Habits

The overall goal of the *Connected Mathematics* curriculum is to help students develop sound mathematical habits. Through their work in this and other algebra units, students learn important questions to ask themselves about situations that can be modeled mathematically.

- *What are the variables in the problem?*
- *How are these variables related to each other?*
- *Which variables depend on, or change in relation to, others?*
- *How can the relationship be displayed and analyzed with tables, graphs and equations?*
- *What does it mean when we see regular and predictable changes in a table of data or a graph?*
- *How can we use these predictable changes to solve problems?*

Mathematics of the Unit

Overview

Situations that change are a part of everyone's life. Some situations change in predictable patterns. Others change in ways that seem beyond our ability to anticipate. It is human nature to want to analyze, anticipate, and predict how things change. Learning to observe, describe, and record changes is the first step in analyzing and searching for patterns in a real-world situation.

The central theme of the *Connected Mathematics* approach to algebra is the importance of studying relationships among quantitative variables. Those relationships are commonly represented in mathematics by tables of (x, y) data pairs, by graphs of related data values, by statements that describe (in words) the way the variables are related, and by equations that describe the relationship between x and y in compact symbolic form.

Variables and Patterns, the first unit of the *Connected Mathematics* algebra strand, develops students' ability to explore a variety of situations in which changes occur. In the first part of the unit, students explore three ways of representing a changing situation: in the narrative, with a data table, and with a graph. These three methods of organizing and recording data are revisited throughout the unit. They are compared to one another to elicit the strengths of each presentation.

In practice, effective use of both tables and graphs requires mathematical judgments. In the case of tables, one has to decide which values of the independent variable should be represented in the table to give most informative results. Should the value of the independent variable be shown in increments of 1, 2, 5, or 10? How far should the table extend?

In the case of graphs, one has to identify the independent and dependent variables in the relationship being studied, make choices of appropriate scales on the horizontal and vertical axes, plot a sample of (x, y) data points, and make decisions about whether and how those data points should be connected.

Effective use of graphs also requires the ability to "read" the numerical story from the shape of a function graph. For graphs that represent change over time, interpretation requires understanding that as one moves along the graph from left to right, a rising graph means an increasing value of the dependent variable, while a falling graph means a decreasing value of the dependent variable.

Later in the unit, students begin to write symbolic equations as a shorter, quicker way to give a summary of the relationship between two variables. This work comes only after extensive time and effort are devoted to analyzing data sets showing change and describing the change in words. The advantages of an equation over a data table or graph are investigated. Students do not learn systematic strategies to solve equations in this unit. However, they do use informal reasoning to find values for x or y given a value for the other. After becoming comfortable in writing equations, students learn how to use graphing calculators to make tables and graphs for any given equation.

The organization of this unit reflects the growing body of experience in function-oriented approaches to algebra, which suggests that students are more comfortable studying quantitative relationships if they work first from numerical data, usually displayed in tabular form. The unit interweaves graphs, tables, verbal descriptions, and equations to support students who are more comfortable with particular forms of representation. However, the goal is for all students to make progress in understanding and being able to think and reason with all major useful forms of representation.

Summary of Investigations

Investigation 1

Variables, Tables, and Coordinate Graphs

Investigation 1 introduces the idea of a variable and three different ways to represent relationships between variables: verbal descriptions, tables, and graphs. The context for the problems is the planning of a bike tour by a group of college students. Students interpret and create data representations and begin to think about the strengths and weaknesses of each type of representation.

Investigation 2

Analyzing Graphs and Tables

Investigation 2 focuses on making and using tables and graphs to help make decisions about costs and profit for the bike tour. The last problem involves matching verbal descriptions with related graphs.

Investigation 3

Rules and Equations

Investigation 3 develops strategies for writing symbolic equations, or formulas, to represent relationships between quantitative variables. Students first write equations involving one operation, and then move to two-operation equations. The last problem involves writing equations for revenue, expenses, and profit for the bike tour.

Investigation 4

Calculator Tables and Graphs

Investigation 4 has two problems that help students learn to make and use tables and graphs on a graphing calculator. The last problem allows students to review the strategies and techniques developed in Investigations 1–3 and compares their own work with tables and graphs generated on a graphing calculator.

Mathematics Background

Algebra

Through their work in this unit, your students learn that a variable is a quantity that might assume many different values. Students work on problems that require them to predict the pattern of change in a variable as time passes or to predict the way changes in values of one variable are related to changes in values of another. This unit introduces some basic tools of algebra. Students are not expected to develop a complete understanding of algebraic ideas. These ideas will be revisited and further developed in each of the succeeding algebra units.

Verbal Descriptions

Verbal descriptions of a relationship are useful because they are descriptions in students' everyday language. This helps students form mental pictures of the situations and the relationships among the variables. The disadvantage of verbal descriptions is that they are sometimes ambiguous, making it difficult to get a quick overview of the situation and the relationships among variables.

Tables

Tables are usually easy to read. From a table, you can see how a change in one variable affects the value of the other variable. Students can recognize whether the change is additive, multiplicative, or unpredictable. Once students recognize the pattern of change, they can extend the pattern to get the next entry. For example, consider these tables.

Table 1: Linear Relationship

x	0	1	2	3
y	5	6	7	8

Table 1: Exponential Relationship

x	0	1	2	3
y	1	2	4	8

In Table 1, as the values of the variable x change by 1 unit, the values of y change by 1 unit. Adding 1 to the previous entry in the x column and 1 to the previous entry in the y column can continue the table. If x is 3, then y is $7 + 1$, or 8. The particular change pattern in Table 1 is indicative of all *linear relationships*.

The change pattern in Table 2 is characteristic of *exponential relationships*. It is a multiplicative pattern because the values of y double, or increase by a factor of 2, as the values of x increase by 1 unit.

In some tables, the patterns of change are not regular. For example, Table 3, which occurs in the first investigation, does not show a pattern of change that is regular; that is, there is no way to predict the change from one point to the next.